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Gravity localization with a domain wall junction in six dimensions

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Abstract

We study gravity localization in the context of a six-dimensional gravity model coupled with complex scalar fields. With a supergravity-motivated scalar potential, we show that the domain wall junction solutions localize a four-dimensional massless graviton under an assumption on the wall profile. We find that unlike the global supersymmetric model, contributions to the junction tension cancel locally with gravitational contributions. The wall tension vanishes due to the metric suppression.

1 Introduction

Possibilities of extra dimensions have been studied for a long time, and the existence is well motivated by superstring theories. A traditional method to hide extra dimensions is an idea of compactification in which the extra dimensions are supposed to be extremely small.

In a couple of years, it has been recognized that the extra dimensions may have submillimeter size [1] or infinitely large volume [2][3][4] if the standard model fields are confined at a three brane. In particular Randall and Sundrum recently proposed an scenario alternative to compactification using exponentially warped metric in a five-dimensional gravity model [2][3]. Due to the warped metric, the four-dimensional massless graviton are localized on a brane, and the four-dimensional Newton law is approximately realized on the brane at low energy. Supersymmetrization of this scenario has been discussed in Ref. [5]

In the Randall-Sundrum scenario, cosmological constants are introduced for both the five-dimensional bulk and the four-dimensional branes, and the warped metric is derived as a solution to the Einstein equation [6]. In order to obtain the solution with the four-dimensional Poincaré invariance, however, the cosmological constants have to be specially tuned. Stabilization mechanism of the extra dimension has been discussed by introducing a bulk scalar field [7].

In order to have a natural explanation of this tuning, we have to discuss the origin of the brane cosmological constant. Instead of pursuing string theories, we consider a the field theoretic approach in this paper. Several works using domain wall solutions in five-dimensional gravity models have been done [8][9][10][11]. In these analyses, a supergravity-motivated scalar potential is introduced [12], and the domain wall solution of the scalar fields produces an effective cosmological constant on the brane to implement the warped metric in the Randall-Sundrum scenario. This line is a gravity version of a previous idea of living in a domain wall [13][14]. An analysis on domain walls in arbitrary dimensions is given in Ref. [15]. Domain wall solutions in four-dimensional supergravity models have been studied in Ref. [16].

However it has recently been pointed out that smooth domain wall solutions interpolating between supersymmetric vacua can not exist in odd-dimensional supergravity models [17]. This naturally leads us to work in a framework of six-dimensional supergravity models [18]. In this case we need a two-dimensional topological solution like a vortex solution [19]. Gravity localization on a string-like defect in six dimensions has been studied in Ref. [20].

There is another interesting two-dimensional stable solitonic solution, namely, a domain wall junction solution in supersymmetric models [21][22][23][24][25]. The domain wall junction preserves one-quarter of the underlying supersymmetry [22][23], and satisfies a first order BPS equation [26]. With domain wall junctions, general analyses on gravity localization in infinitely large extra dimensions have been given in Refs. [27][28][29].

In this paper we study gravity localization in the context of a six-dimensional gravity model coupled with complex scalar fields. Similar analyses have been done in Refs. [30][31]. With a supergravity-motivated scalar potential, we derive first order equations which the metric and the scalar fields should satisfy. We calculate the tensions of domain wall and juncion. Finally we study a warped metric and discuss gravity localization on the junction.

2 Set-up

We consider a six-dimensional gravity coupled with complex scalar fields ϕ^i ($i = 1, \cdots, N$). We use coordinates $x^M = (x^\mu, x^m)$, where $M = 0, 1, 2, 3, 5, 6$, $\mu = 0, 1, 2, 3$ and $m = 5, 6$. x^μ are coordinates for the observable four spacetime dimensions. x^m are coordinates for the two extra dimensions, where $-\infty < x^m < \infty$. The action is given by

$$S = \int d^6x\sqrt{-g}\left[-\frac{1}{2\kappa^2}R + K_{ij^*}g^{MN}\partial_M\phi^i\partial_N\phi^{j*} - V(\phi, \phi^*)\right], \tag{1}$$

where g_{MN} is the metric in a time-like signature convention $(+, -, -, -, -, -)$. $V(\phi, \phi^*)$ is a scalar potential. The scalar kinetic term has field dependent coefficients $K_{ij^*}(\phi, \phi^*)$ which are derived from the Kähler potential as $K_{ij^*} = \partial^2 K(\phi, \phi^*)/\partial\phi^i\partial\phi^{j*}$. In the following, we adopt the six-dimensional Planck mass unit $\kappa^2 = 1$ unless otherwise stated. We put the following ansatz for the background metric

$$\begin{aligned} ds^2 &= g_{MN}dx^M dx^N \\ &= e^{2\sigma(x^5, x^6)}\left[\eta_{\mu\nu}dx^\mu dx^\nu - (dx^5)^2 - (dx^6)^2\right], \end{aligned} \tag{2}$$

where $\eta_{\mu\nu}$ is a four-dimensional flat metric. This metric guarantees the four-dimensional Poincaré invariance. We look for static scalar configurations which depend on x^m only:

$\phi^i = \phi^i(x^5, x^6)$. Then the equations of motion can be written as

$$\begin{aligned} \partial_m^2 \sigma + 4(\partial_m \sigma)^2 &= -\frac{1}{2} e^{2\sigma} V, & (3) \\ -4\partial_m \partial_n \sigma + 4(\partial_m \sigma)(\partial_n \sigma) &= K_{ij^*}(\partial_m \phi^i \partial_n \phi^{j*} + \partial_n \phi^i \partial_m \phi^{j*}), & (4) \\ \partial_m^2 \phi^i + 4(\partial_m \sigma)(\partial_m \phi^i) & \\ + K^{ij^*}(\partial_k K_{lj^*})(\partial_m \phi^k)(\partial_m \phi^l) &= e^{2\sigma} \frac{\partial V}{\partial \phi^{j*}}, & (5) \end{aligned}$$

where repeated indices are summed. The first two equations correspond to (μ, ν) and (m, n) components of the Einstein equations, respectively. The last one is the scalar field equation.

For the scalar potential, we assume the following form motivated by supergravity models [8][15][32]:

$$V = e^K \left[K^{ij^*}(D_i W)(D_j W)^* - \frac{5}{2}|W|^2 \right], \tag{6}$$

where $D_i W = \partial W / \partial \phi^i + (\partial K / \partial \phi^i) W$ and W is an arbitrary function of ϕ^i which may be interpreted as a superpotential if we can derive this potential from a supergravity model. A five-dimensional gauged supergravity model has a similar potential [12]. Given the potential (6), we can obtain the first order equations which the metric and the scalar fields should satisfy [9][8][11][15][30][31]. We will see this in the next section. These first order equations make it easy to solve the equations of motion (3)-(5).

In general the potential (6) is not bounded below because of $|W|^2$ term. Furthermore in some higher-dimensional supergravity models, there are no local minima and all the extrema are local maxima or saddle points. However it has been shown that even a vacuum at a maximum is stable under local fluctuations around an AdS background unless the curvature of the potential at the maximum is too negative [32]. In this sense the potential (6) is sensible.

No six-dimensional supergravity models which provides the potential (6) have been constructed, so we can not work in a supergravity context. In this paper we just assume the potential (6) from the beginning.

3 Tensions of domain wall and junction

In this section, we derive the first order equations for the metric and the scalar fields. We also obtain the formula for the tensions of domain wall and junction.

With the potential (6), we can write the action as a sum of perfect squares up to total derivative terms [26]. For this purpose, it is convenient to define a complex coordinate variable $z = x^5 + ix^6$ and derivatives $\partial = (\partial_5 - i\partial_6)/2$, $\bar{\partial} = (\partial_5 + i\partial_6)/2$. In terms of this variable, the action (1) is written as

$$\begin{aligned} S &= \int d^6x\, e^{6\sigma} \Big[-20e^{-2\sigma}\left(\bar{\partial}\partial\sigma + 2\partial\sigma\bar{\partial}\sigma\right) - 2e^{-2\sigma}K_{ij^*}\left(\partial\phi^i\bar{\partial}\phi^{j*} + \bar{\partial}\phi^i\partial\phi^{j*}\right) \\ &\qquad -e^K\left\{K^{ij^*}(D_iW)(D_jW)^* - \tfrac{5}{2}|W|^2\right\}\Big]. \end{aligned} \tag{7}$$

We can make a perfect square, e.g., from a part of the $\partial\sigma\bar{\partial}\sigma$ term and the $|W|^2$ term. In doing that we can introduce a complex phase $\theta(x^5, x^6)$ as follows:

$$\left|\partial\sigma - \tfrac{1}{4}e^\sigma e^{K/2}e^{i\theta}W^*\right|^2. \tag{8}$$

Taking into account this phase, the action can be written as a sum of a local contribution and a topological term [15][26][30]

$$S = S_{\rm local} + S_{\rm topological}. \tag{9}$$

The local contribution is given by a sum of perfect squares

$$\begin{aligned} S_{\rm local} &= \int d^6x\, e^{4\sigma}\left[40|D\sigma|^2 - 4K_{ij^*}(D\phi^i)(D\phi^j)^* - \{4i(D\theta)(D\sigma)^* + \text{h.c.}\}\right] \\ &= \int d^6x\, e^{4\sigma}\left[40\left|D\sigma - \tfrac{i}{10}D\theta\right|^2 - 4K_{ij^*}(D\phi^i)(D\phi^j)^* - \tfrac{2}{5}\left|D\theta\right|^2\right], \end{aligned} \tag{10}$$

where the symbols $D\sigma$, $D\phi^i$ and $D\theta$ are defined by

$$D\sigma = \partial\sigma - \tfrac{1}{4}e^\sigma e^{K/2}e^{i\theta}W^*, \tag{11}$$

$$D\phi^i = \partial\phi^i + \tfrac{1}{2}K^{ij^*}e^\sigma e^{K/2}e^{i\theta}(D_jW)^*, \tag{12}$$

$$D\theta = \partial\theta - \tfrac{i}{2}\left(K_i\partial\phi^i - K_{i^*}\partial\phi^{i*}\right). \tag{13}$$

The perfect squares in the integrand imply that the configurations which extremize the action satisfy the first order equations $D\sigma = D\phi^i = D\theta = 0$, namely,

$$\partial\sigma = \frac{\kappa^2}{4}e^\sigma e^{\kappa^2K/2}e^{i\theta}W^*, \tag{14}$$

$$\partial\phi^i = -\frac{1}{2}K^{ij^*}e^{\sigma}e^{\kappa^2 K/2}e^{i\theta}(D_jW)^*, \tag{15}$$

$$\partial\theta = \frac{i}{2\kappa^2}\left(K_i\partial\phi^i - K_{i^*}\partial\phi^{i*}\right). \tag{16}$$

Here we have recovered the gravitational coupling explicitly. Note that the solutions to equations (14)-(16) automatically satisfy the equations of motion (3)-(5). In the four-dimensional supergravity model, similar equations are derived from conditions for the existence of unbroken supersymmetry. The equation (15) implies that the scalar fields ϕ^i should depend on both z and z^* in order to have a non-trivial configulation.

Because of the negative signs in front of the perfect squares in eq. (10), the solutions to these first order equations do not minimize the energy in general. However on an AdS background, the vacuum at an extremum is stable under local fluctuations unless the curvature of the potential at the maximum is too negative [32].

The equation (16) can be written as $\partial_m\theta = -\mathrm{Im}(K_i\partial_m\phi^i)$. In the four dimensional supergravity model, the quantity $\mathrm{Im}(K_i\partial_m\phi^i)$ corresponds to an auxiliary vector field in a supergravity multiplet. This field acts as a gauge field for the Kähler-Weyl invariance.

The topological term consists of two kinds of total derivative terms

$$S_{\text{topological}} = -\int d^4x \sum_m (Z_m + Y_m). \tag{17}$$

The first contributions Z_m are given by

$$\begin{aligned} Z_5 &= \int dx^5 dx^6\, \partial_5 \left[e^{4\sigma}(5\partial_5\sigma) - e^{5\sigma}e^{K/2}(e^{-i\theta}W + e^{i\theta}W^*)\right], \\ Z_6 &= \int dx^5 dx^6\, \partial_6 \left[e^{4\sigma}(5\partial_6\sigma) + ie^{5\sigma}e^{K/2}(e^{-i\theta}W - e^{i\theta}W^*)\right]. \end{aligned} \tag{18}$$

These include the domain wall tensions and the metric terms. The second contributions are

$$\begin{aligned} Y_5 &= -\int dx^5 dx^6\, \partial_5 \left[e^{4\sigma}\left\{\partial_6\theta + \mathrm{Im}(K_i\partial_6\phi^i)\right\}\right], \\ Y_6 &= \int dx^5 dx^6\, \partial_6 \left[e^{4\sigma}\left\{\partial_5\theta + \mathrm{Im}(K_i\partial_5\phi^i)\right\}\right]. \end{aligned} \tag{19}$$

These include the ordinary domain wall junction tentions $\mathrm{Im}(K_i\partial_m\phi^i)$ and 'gravitational' contributions $\partial_m\theta$. Note that Y_m can have non-vanishing values only when the fields have two-dimensional non-trivial configurations, while Z_m can be non-vanishing even when the fields depends on only one of x^m.

In the absence of gravity, the derivatives of θ do not contribute, and the terms $\mathrm{Im}(K_i\partial_m\phi^i)$ give the domain wall junction tension. Note that $\partial_m\theta$ terms in eq. (19) are proportional to κ^2 if we write the gravitational coupling explicitly. In the presence of gravity, however, the derivatives of θ contribute to Y_m. Substituting the first order equation (16) into eq. (19), we find that the two contributions $\partial_m\theta$ and $\mathrm{Im}(K_i\partial_m\phi^i)$ cancel locally in the integrand with each other. Therefore the junction tentions Y_m vanish

$$Y_m \;=\; 0. \tag{20}$$

This implies that in the presence of the gravitational degrees of freedom, a constant θ configulation can not extremize the action. The system requires a non-trivial θ to extremize the action in such a way that $\partial_m\theta$ cancel the ordinary contributions $\mathrm{Im}(K_i\partial_m\phi^i)$ to the junction tension.

Let's explain briefly the reason why $\partial_m\theta$ and $\mathrm{Im}(K_i\partial_m\phi^i)$ appear in the vanishing combination $D\theta$. In order to see this, it is essential to observe that the total derivative in the domain wall tension term produce the vanishing combination as follows:

$$\partial_m\left[e^{K/2}e^{-i\theta}W\right] \;=\; e^{K/2}e^{-i\theta}\left[(D_iW)\partial_m\phi^i - iW(\partial_m\theta + \mathrm{Im}K_i\partial_m\phi^i)\right]. \tag{21}$$

We can see that in the other parts of calculations, $\partial_m\theta$ and $\mathrm{Im}(K_i\partial_m\phi^i)$ always appear in the same combination. If we work in the four-dimensional supergravity model, we may have deeper understanding of this result referring to the Kähler-Weyl invariance.

As for the wall contributions Z_m, the situation is different. Substituting the first order equation (14) into eq. (18), we see that Z_m are written as

$$\begin{aligned} Z_5 &= \int dx^5dx^6\,\partial_5\left[\tfrac{1}{2}e^{5\sigma}e^{K/2}\mathrm{Re}(e^{-i\theta}W)\right], \\ Z_6 &= \int dx^5dx^6\,\partial_6\left[\tfrac{1}{2}e^{5\sigma}e^{K/2}\mathrm{Im}(e^{-i\theta}W)\right]. \end{aligned} \tag{22}$$

Unlike the junction tensions, the wall tensions Z_m does not vanish in general.

4 Warped metric from domain wall junction

In this section, we discuss the solution to the first order equations (14)-(16). In most cases the analytic solution is not available, so in this papar we only discuss rough behavior of the solution. As an example for the function W, let's consider a quartic form for a single complex scalar field ϕ

$$W = -\frac{1}{4}\phi^4 + \phi. \tag{23}$$

For the Kähler potential K, we take the minimal form $K = \phi\phi^*$. It is known that in the global supersymmetric model such a quartic superpotential W allows static domain wall junction solutions [21][22][23][24]. In the global case, the scalar potential has three isolated degenerate minima at $\phi = 1,\, \omega,\, \omega^2$, where $\omega = e^{2\pi i/3}$. Note that the potential does not allow static domain wall solutions in the four-dimensional supergravity model [16]. This comes from the reality of the metric. In the domain wall junction case, however, the same discussion cannot be applied since the first order equation (14) for the metric includes the derivative with respect to the complex coordinate z.

With the function W in eq. (23) and the minimal Kähler potential K, we find three vacua $\phi = k,\, k\omega,\, k\omega^2$ by solving $D_\phi W = 0$. Here $k \approx 1.2$ is a single solution of a quintic equation $k^5 + 4k^3 - 4k^2 - 4 = 0$. The potential (6) and the first order equations (14)-(16) in this case are invariant under $\mathbf{Z}_3$ action $z \to \omega z$, $\phi \to \omega\phi$, $\sigma \to \sigma$, $\theta \to \theta$. Therefore we expect a $\mathbf{Z}_3$ invariant domain wall junction solution. The solution describes the two-dimensional space separated into three regions by three walls, and these regions are labeled by the vacua $k,\, k\omega,\, k\omega^2$. Equations (15) and (16) imply that outside the walls and the junction the scalar fields ϕ and the phase θ are almost constant. We consider the solution with $\arg\phi = \arg(-z)$ outside the wall so that the junction is centered at $z = 0$. Then the first order equation (14) leads to the asymptotic behavior of the metric at spacial infinity in the extra dimensions

$$e^{2\sigma} \longrightarrow \frac{C^2}{[\mathrm{Re}(\omega^{*n} z)]^2}, \qquad |z| \longrightarrow \infty, \tag{24}$$

where $n = 0,\, 1,\, 2$ label the vacua $k,\, k\omega,\, k\omega^2$, respectively. The constant C is given by $C = 2e^{-k^2}(k - k^4/4)^{-1}$, where we have chosen $\theta = 0$ outside the wall. The above metric (24) describes an AdS background. This asymptotic form is consistent with analyses in Refs. [27][28][30]. From this behavior it follows that the extra dimension is infinitely large, since the volume $V = \int dx^5 dx^6\, e^{2\sigma} \sim \int dr/r$ is divergent where $r = |z|$.

Note that this behavior (24) can be applied only outside the walls. Inside the wall the scalar field is not constant any more, hence we have to solve the coupled equations (14)-(16) to know the profile. However it seems natural to assume that for a fixed r, the wall essentially has a kink-like profile. Then the evolution equation (15) along the direction perpendicular to the wall is given by $\partial\phi \sim r^{-1}\, (D_\phi W)^*$ inside the wall. This is consistent with the metric behavior (24) outside the wall. The metric may have a peak structure in the wall [16], but the height of the peak scales as $1/r^2$. Thus the scaling property $e^{2\sigma} \sim 1/r^2$ holds even in the walls. Also this assumption means that the wall width grows up like $\sim r$ far from the origin. On the other hand the 'height' of the wall remains the same even far from the origin. Consequently, the wall structure almost disappears far from the junction due to the metric suppression.

It is difficult to discuss the solution inside the junction, too. However we can see the behavior of the metric near the origin $x^m = 0$ in a weak gravitational coupling approximation. In the $\kappa^2 = 0$ limit, the metric σ and the phase θ are constant, and eq. (15) reduces to a simpler equation. From symmetry consideration the scalar field must vanish at the origin. Then it follows that $\phi \approx -z$ from eq. (15). In the first order in κ^2, the metric equation (14) becomes $\partial\sigma = W^*/4$ where we have substituted the zeroth order result in the right-hand side. Using this equation, we obtain $\sigma \approx -zz^*/4$ near the origin so that

$$e^{2\sigma} \approx 1 - \frac{zz^*}{2}, \qquad |z| \ll 1. \tag{25}$$

The curvature near the origin is constant $R \approx -10$ up to the order of $|z|^2$.

Let's estimate the topological charges Z_m in this example. Unlike the case of Y_m, the integrand of Z_m in eq. (22) does not vanish locally. In this example, however, the integrand goes to zero at spatial infinity in the extra dimensions faster than r due to the asymptotic power law suppression of the metric (24). Therefore, using the Stokes's theorem, the domain wall tensions also vanish

$$Z_m = 0. \tag{26}$$

Given $Y_m = 0$ in eq. (20), we find that all the topological contribution to the action vanish. Namely, the four-dimensional cosmological constant is zero. This is consistent with our ansatz of the flat four-dimensional spacetime in eq. (2).

The discussion here is based on a qualitative consideration. In particular the behavior of the solution inside the walls and the junction has not been obtained. In order to derive more precise information, it seems helpful to study numerically.

5 Gravity localization

In this section we study the fluctuations of the metric defined by

$$ds^2 = g_{MN}dx^M dx^N + h_{\mu\nu}dx^\mu dx^\nu, \tag{27}$$

where the first term in the right-hand side is the background metric in eq. (2). In this analysis, we focus on the transverse traceless modes which satisfy $h^{\mu}{}_{\mu} = \partial^{\mu} h_{\mu\nu} = 0$.

In order to obtain the linealized equation for the fluctuation, we expand the action (1) in terms of $h_{\mu\nu}$ around the background metric up to the second order of the fluctuations. The second order terms are given by

$$\begin{aligned} S^{(2)} &= \int d^6 x \left[-\tfrac{1}{8} h^{\mu\nu} \left\{ \Box_4 - \partial_m^2 + 2(\partial_m^2 \sigma) + 4(\partial_m \sigma)^2 \right\} h_{\mu\nu} \right. \\ &\quad \left. + \left\{ \partial_m^2 \sigma + \tfrac{3}{2} (\partial_m \sigma)^2 + \tfrac{1}{4} (K_{ij^*} \partial_m \phi^i \partial_m \phi^{j*} + e^{2\sigma} V) \right\} h^{\mu\nu} h_{\mu\nu} \right], \end{aligned} \tag{28}$$

where upper Lorentz indices are defined by the flat metric $\eta_{\mu\nu}$. The terms in the second curly bracket in the right-hand side cancel because of the first order equations (14)-(16). Variation of $S^{(2)}$ with respect to $h_{\mu\nu}$ gives rise to the linearized equation of motion for the metric fluctuation

$$\left[\Box_4 - \partial_m^2 + 2(\partial_m^2 \sigma) + 4(\partial_m \sigma)^2 \right] h_{\mu\nu} = 0. \tag{29}$$

For the fluctuation with the four-dimensional dependence of a plane wave e^{ipx}, this equation can be written as a two-dimensional Schrödinger equation $(-\partial_m^2 + V_{\rm QM})\, h_{\mu\nu} = p^2 h_{\mu\nu}$ with a potential

$$V_{\rm QM} \;=\; 2(\partial_m^2 \sigma) + 4(\partial_m \sigma)^2. \tag{30}$$

In the following we again consider the quartic function W in eq. (23) and the minimal Kähler potential. Using eq. (24), the asymptotic behavior of this potential far from the junction is given by

$$V_{\rm QM} \;\longrightarrow\; \frac{6}{[{\rm Re}(\omega^{*n} z)]^2}. \tag{31}$$

In five-dimensional models, the solutions to the corresponding Schrödinger equation for $p^2 > 0$ are described by the Bessel functions $\sqrt{x} J_2(px)$ and $\sqrt{x} Y_2(px)$ [3][9]. In our case, the equation involves the two variables, and the complete solution is not available. However the potential (31) shows that outside the wall, the Schrödinger equation is essentially one dimensional equation. Therefore the solutions for $p^2 > 0$ can be described by the Bessel functions $\sqrt{x} J_{5/2}(px)$ and $\sqrt{x} Y_{5/2}(px)$ outside the wall, where $x = {\rm Re}(\omega^{*n} z)$. These can be written by trigonometric functions only. The similar solutions have been found in Ref. [20].

In the case of the massless fluctuation $p^2 = 0$, the solution to eq. (29) is given by [30]

$$h_{\mu\nu} = e^{2\sigma} e^{ipx} \eta_{\mu\nu}. \tag{32}$$

This means that the massless fluctuation has the same configuration as the background metric in the extra dimensions so that the massless graviton is localized on the junction. Ihis agrees with the result in Ref. [30]. From eq. (24), we see that the massless mode (32) is normalizable on our curved background

$$\int dx^5 dx^6 \, e^{2\sigma} h^{\mu\nu} h_{\mu\nu} \ \ < \ \ \infty. \tag{33}$$

In the transverse traceless components which we have considered, there is no tachyonic mode. This is shown by writing eq. (29) as follows [9]

$$Q_m^\dagger Q_m h_{\mu\nu} = p^2 h_{\mu\nu}, \tag{34}$$

where $Q_m \equiv -\partial_m + 2(\partial_m \sigma)$. In the flat space, $Q_m^\dagger \equiv \partial_m + 2(\partial_m \sigma)$ is the adjoint of Q_m. Similar equation appears in supersymmetric quantum mechanics. The solutions to the Schrödinger equation for $p^2 < 0$ are described by the modified Bessel functions $\sqrt{x} I_{5/2}(px)$ and $\sqrt{x} K_{5/2}(px)$. Following the discussion in Ref. [9], we can see that normalizable modes always satisfy $p^2 \geq 0$ for the transverse traceless components.

6 Summary

In summary, we have studied a six-dimensional gravity coupled with complex scalar fields. With a supergravity-motivated scalar potential, the domain wall junction solutions localize a four-dimensional massless graviton. We have shown that unlike the global supersymmetric model, contributions to the junction tension cancel locally with gravitational contributions. The wall tension vanishes due to the metric suppression.

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